Period 3, Jan 22, 2025

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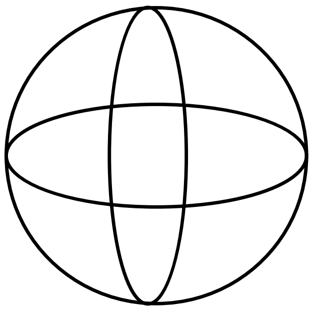


$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dT} = 650 \ \text{cm}^3/\text{min}$

$\frac{dV}{dT} = 4\pi r^2 \frac{dr}{dT}$

$r = 16$

$650 \ \text{cm}^3/\text{min} = 4 \cdot \pi \cdot 16^2 \frac{dr}{dT}$

$\frac{650}{1024\pi} = \frac{\cancel{1024\pi}}{\cancel{1024\pi}} \frac{dr}{dT}$

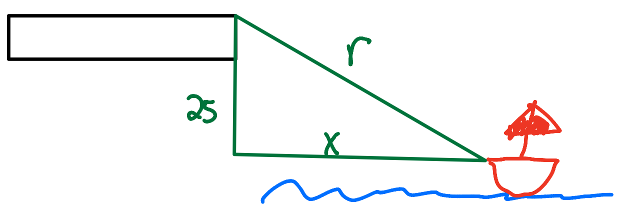
$\frac{325}{512\pi} \ \text{cm/min} = \frac{dr}{dT}$

$r = 15$

$650 \ \text{cm}^3/\text{min} = 4 \cdot \pi \cdot 15^2 \frac{dr}{dT}$

$\frac{650}{900\pi} = \frac{\cancel{900\pi}}{\cancel{900\pi}} \frac{dr}{dT}$

$\frac{13}{18\pi} \ \text{cm/min} = \frac{dr}{dT}$



both

$25^2 + x^2 = r^2$

$625 + x^2 = r^2$

$0 + 2x\frac{dx}{dT} = 2r\frac{dr}{dT}$

$r = 45$

$25^2 + x^2 = 45^2$

$x = \sqrt{1400} = 10\sqrt{14}$

$\frac{dr}{dT} = -14 \ \text{FT/sec}$

$2x\frac{dx}{dT} = 2r\frac{dr}{dT}$

$2 \cdot 10\sqrt{14} \cdot \frac{dx}{dT} = 2 \cdot 45 \cdot -14$

$\frac{dx}{dT} = \frac{-630}{10\sqrt{14}} = \frac{-63\sqrt{14}}{14} = -\frac{9\sqrt{14}}{2} \ \text{FT/s}$

$\frac{dr}{dT} = -10 \ \text{FT/sec}$

$2x\frac{dx}{dT} = 2r\frac{dr}{dT}$

$2 \cdot 10\sqrt{14} \frac{dx}{dT} = 2(45)(-10)$

$\frac{dx}{dT} = \frac{-45}{\sqrt{14}} = \frac{-45\sqrt{14}}{14} \ \text{FT/s}$

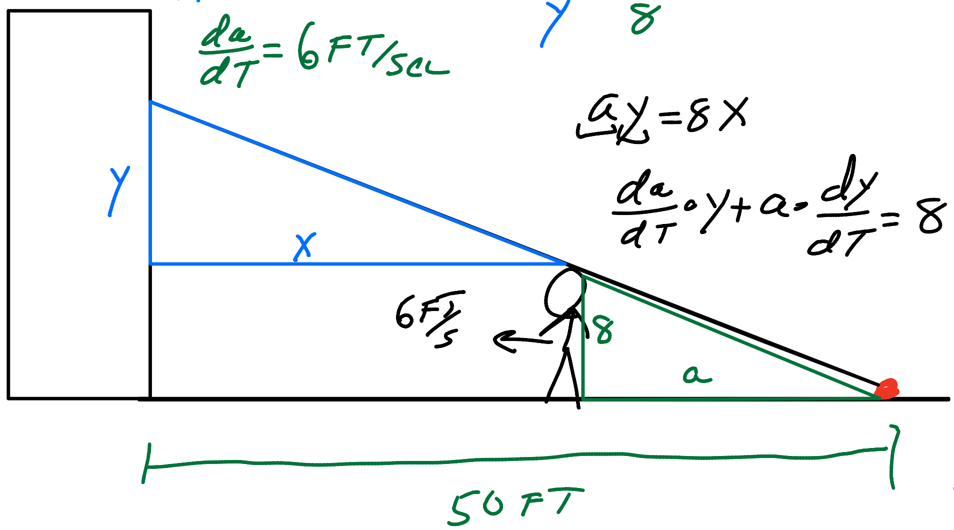
$$\frac{dx}{dT} = -6\,FT/sec$$

$$\frac{da}{dT} = 6\,FT/sec$$

$$\frac{x}{y} = \frac{a}{8}$$

$$ay = 8x$$

$$\frac{da}{dT}\cdot y + a\cdot\frac{dy}{dT} = 8\,\frac{dx}{dT}$$



$$\frac{x}{y} = \frac{a}{8}$$

$$\frac{20}{y} = \frac{30}{8}$$

$$30y = 160$$

$$y = \frac{16}{3}$$

Find $\frac{dy}{dT}$ when $x = 20 \Rightarrow a = 30$

$$6\cdot y + 30\cdot\frac{dy}{dT} = 8\cdot -6$$

$$6\cdot\frac{16}{3} + 30\cdot\frac{dy}{dT} = -48$$

$$32 + 30\,\frac{dy}{dT} = -48$$

$$-32 \qquad\qquad -32$$

$$\frac{\cancel{30}\,\frac{dy}{dT}}{\cancel{30}} = \frac{-80}{30} \Rightarrow \frac{dy}{dT} = -\frac{8}{3}\ FT/s$$

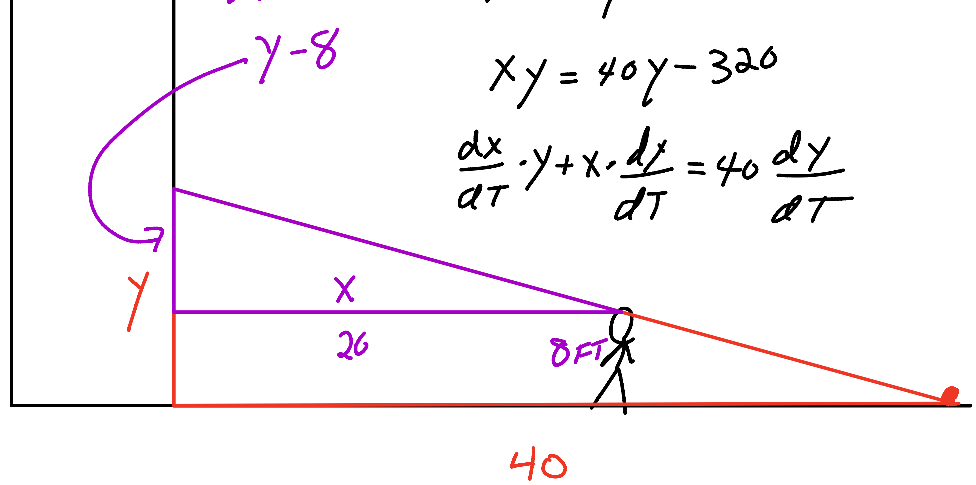
Find $\frac{dy}{dT}$ when $X=20$

$\frac{dx}{dT} = -7 FT/S$

$\frac{X}{y-8} = \frac{40}{y}$

$xy = 40y - 320$

$\frac{dx}{dT} \cdot y + x \cdot \frac{dy}{dT} = 40 \frac{dy}{dT}$



$X=20$

$20y = 40y - 320$

$-20y = -320$

$y = 16 FT$

$-7 \cdot y + 20 \frac{dy}{dT} = 40 \frac{dy}{dT}$

$-20 \frac{dy}{dT} \qquad -20 \frac{dy}{dT}$

$-7y = 20 \frac{dy}{dT}$

$\frac{-28}{5} = \frac{-112}{20} = \frac{-7 \cdot 16}{20} = \frac{\cancel{20} \frac{dy}{dT}}{\cancel{20}}$

$\frac{-28}{5}$ FT/sec

d) $f(t) = 2t - \cos t + \frac{1}{t}$ $= \int (2T - \cos T + \frac{1}{T}) dT =$

$2 \cdot \frac{1}{2} \cdot T^{1+1=2} - \sin T + \ln|T| + C = T^2 - \sin T + \ln|T| + C$

$\frac{1}{3}+1=\frac{4}{3}$

$\frac{3}{2}+1=\frac{5}{2}$

c) $\frac{dy}{dx}=x^{1/3}+x\sqrt{x}-2$; when $x=1, y=2$

$\int dy=\int (x^{\frac{1}{3}}+x^{\frac{3}{2}}-2)\,dx$

$y=1\cdot\frac{3}{4}\cdot x^{\frac{4}{3}}+1\cdot\frac{2}{5}\cdot x^{5/2}-2x+C$

$y=\frac{3}{4}x^{\frac{4}{3}}+\frac{2}{5}x^{5/2}-2x+C$

$2=\frac{3}{4}(1)^{4/3}+\frac{2}{5}(1)^{5/2}-2(1)+C \Rightarrow 2=\frac{3\cdot5}{4\cdot5}+\frac{2\cdot4}{5\cdot4}-\cancel{2}+C$

$+2 \qquad +2$

$4=\frac{15}{20}+\frac{8}{20}+C$

$4=\frac{23}{20}+C$

$\frac{80}{20}=\cancel{\frac{23}{20}}+C$

$-\frac{23}{20} \qquad -\frac{23}{20}$

$\frac{57}{20}=C$

$y=\frac{3}{4}x^{4/3}+\frac{2}{5}x^{5/2}-2x+\frac{57}{20}$

d) $f'(x) = x - 2\sin x\,;\ f(\pi) = 0$

$F(x) = \int (x - 2\sin x)\,dx$

$F(x) = \frac{1}{2}x^2 - 2(-\cos x) + C$

$0 = \frac{1}{2}\pi^2 + 2\cos\pi + C$

$0 = \frac{1}{2}\pi^2 + 2(-1) + C$

$-\frac{1}{2}\pi^2 + 2 = C$

$F(x) = \frac{1}{2}x^2 + 2\cos x - \frac{\pi^2}{2} + 2$

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e) $\frac{d^2y}{dx^2} = e^x$; when $x = 0, y = 2,$ when $x = 1, y = e$

$\frac{dy}{dx} = \int e^x\,dx$

$\frac{dy}{dx} = e^x + C_1$

$y = \int (e^x + C_1)\,dx$

$y = e^x + C_1x + C_2$

$2 = e^0 + C_1(0) + C_2$

$2 = 1 + 0 + C_2$

$1 = C_2$

$e = e^1 + 1C_1 + 1$

$\cancel{e} = \cancel{e} + C_1 + 1$

$-e \quad -e$

$0 = 1 + C_1$

$-1 = C_1$

$y = e^x - 1x + 1$

3. A particle in rectilinear motion moves along the x-axis with velocity $v(t) = t^2 + t, t \geq 0$. If the particle is at $x = -1$ when $t = 0$, then what is the position x of the particle at time $t = 3$?
(Hint: $s(t)$ and $x(t)$ are *both* commonly used to symbolize position as a function of time, t)

$V(T) = T^2 + T$

$S(T) = \int v(T)\,dT = \int (T^2 + T)\,dT$

$X(T) = S(T) = \frac{1}{3}T^3 + \frac{1}{2}T^2 + C$

$S(0) = -1 = \frac{1}{3}(0)^3 + \frac{1}{2}(0)^2 + C$

$-1 = C$

$S(T) = \frac{1}{3}T^3 + \frac{1}{2}T^2 - 1$

$S(3) = \frac{1}{3}(3)^3 + \frac{1}{2} \cdot 3^2 - 1 = \frac{27 \cdot 2}{3 \cdot 2} + \frac{9 \cdot 3}{2 \cdot 3} - 1 = \frac{54}{6} + \frac{27}{6} - \frac{6}{6} = \frac{75}{6} = \frac{25}{2}$

$12\frac{1}{2}$

4. Given the acceleration function below, use Calculus techniques to find the distance s of the object from the origin under the initial conditions, $s(0) = 0\,ft$ and $v(0) = 5$ ft/s

$a(t) = \sin t\ ft/s^2$

$V(T) = \int a(T)\,dT$

$S(T) = \int V(T)\,dT$

$V(T) = \int \sin T\,dT$

$V(T) = -\cos T + C_1$

$5 = -\cos 0 + C_1$

$5 = -1 + C_1$

$6 = C_1$

$V(T) = -\cos T + 6$

$S(T) = \int (-\cos T + 6)\,dT$

$S(T) = -\sin T + 6T + C_2$

$0 = -\sin 0 + 6(0) + C_2$

$0 = 0 + 0 + C_2$

$0 = C_2$

$S(T) = -\sin T + 6T + 0$

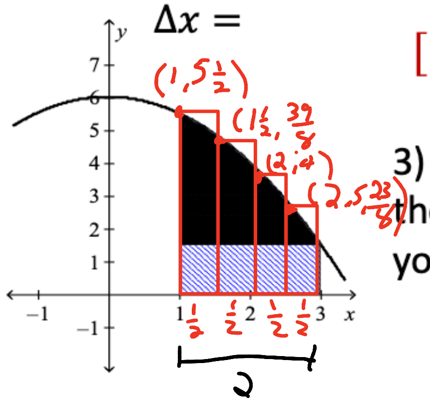
Example 1: Use a Left Riemann Sum with **four** equal rectangles to *approximate* of the area of the region lying between the graph of $f(x) = \left(\frac{-1}{2}\right)x^2 + 6$ and the $\boldsymbol{x}$-axis between $x = 1$ and $x = 3$.

1) Area$\approx$ ___________ + __________ + __________ + ____________

2) The base is _______. So the sub intervals are:

$\Delta x =$

[1,1.5], [1.5,2], [2,2.5], [2.5,3]



3) To find the first approximation use the **left endpoints** as the heights of your rectangles.

$\frac{2}{4}$ = width Rectangle

x	$y = -\frac{1}{2}x^2 + 6$
1	$5\frac{1}{2} = -\frac{1}{2}(1)^2 + 6$
$\frac{3}{2}$	$\frac{39}{8} = -\frac{1}{2}\left(\frac{3}{2}\right)^2 + 6 = \frac{-9}{8} + \frac{48}{8}$
2	$4 \quad -\frac{1}{2}(2)^2 + 6 = -2 + 6 = 4$
$\frac{5}{2}$	$\frac{23}{8} \quad -\frac{1}{2}\left(\frac{5}{2}\right)^2 + 6 = \frac{-25}{8} + \frac{48}{8} = \frac{23}{8}$

$\frac{1}{2} \cdot 5\frac{1}{2} + \frac{1}{2}\left(\frac{39}{8}\right) + \frac{1}{2}(4) + \frac{1}{2}\left(\frac{23}{8}\right)$

$\frac{11}{4} + \frac{39}{16} + \frac{2}{1} + \frac{23}{16} = \frac{44}{16} + \frac{39}{16} + \frac{32}{16} + \frac{23}{16} = \frac{138}{16} = 8.625$

Example 1: Use a ~~Left~~ Right Riemann Sum with **four** equal rectangles to *approximate* of the area of the region lying between the graph of $f(x) = \left(\frac{-1}{2}\right)x^2 + 6$ and the x-axis between $x = 1$ and $x = 3$.

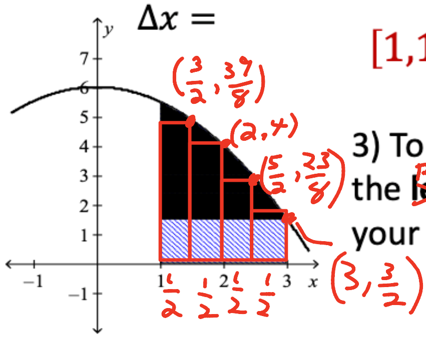
X	Y
1	$5\frac{1}{2} = \frac{11}{2}$
$\frac{3}{2}$	$\frac{39}{8}$
2	4
$2\frac{1}{2}$	$\frac{23}{8}$
3	$\frac{3}{2} = -\frac{1}{2}(3)^2 + \frac{12}{2}$

1) Area≈ __________ + _________ + _________ + ___________

2) The base is _______. So the sub intervals are:

$\Delta x =$

[1,1.5], [1.5,2], [2,2.5], [2.5,3]



3) To find the first approximation use the ~~left~~ Right **endpoints** as the heights of your rectangles.

$$A = \frac{1}{2}\left(\frac{39}{8}\right) + \frac{1}{2}(4) + \frac{1}{2}\left(\frac{23}{8}\right) + \frac{1}{2}\left(\frac{3}{2}\right)$$

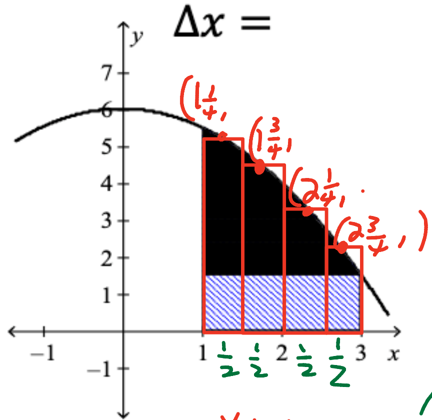
$$\frac{39}{16} + 2 + \frac{23}{16} + \frac{3}{4} = \frac{39}{16} + \frac{32}{16} + \frac{23}{16} + \frac{12}{16} = \frac{106}{16} = 6.625$$

Example 1: Use a ~~Left~~ MidPT Riemann Sum with **four** equal rectangles to *approximate* of the area of the region lying between the graph of $f(x) = \left(\frac{-1}{2}\right)x^2 + 6$ and the x-axis between $x = 1$ and $x = 3$.

1) Area≈ __________ + _________ + _________ + ___________

2) The base is _______. So the sub intervals are:

$\Delta x =$



[1,1.5], [1.5,2], [2,2.5], [2.5,3]

3) To find the first approximation use the ~~left~~ MidPT **endpoints** as the heights of your rectangles.

$$\text{Area} = \frac{1}{2}\left(\frac{167}{32}\right) + \frac{1}{2}\left(\frac{143}{32}\right) + \frac{1}{2}\left(\frac{111}{32}\right) + \frac{1}{2}\left(\frac{71}{32}\right) = \frac{492}{64}$$

$$= 7.6875$$

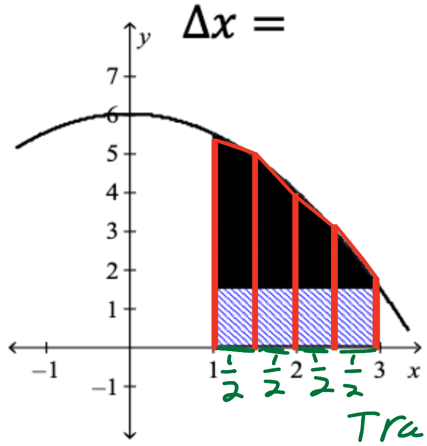
x	y
$1\frac{1}{4} = \frac{5}{4}$	$\left(-\frac{1}{2}\right)\left(\frac{5}{4}\right)^2 + 6 = -\frac{25}{32} + \frac{192}{32} = \frac{167}{32}$
$1\frac{3}{4} = \frac{7}{4}$	$\left(-\frac{1}{2}\right)\left(\frac{7}{4}\right)^2 + 6 = -\frac{49}{32} + \frac{192}{32} = \frac{143}{32}$
$2\frac{1}{4} = \frac{9}{4}$	$\left(-\frac{1}{2}\right)\left(\frac{9}{4}\right)^2 + 6 = -\frac{81}{32} + \frac{192}{32} = \frac{111}{32}$
$2\frac{3}{4} = \frac{11}{4}$	$-\frac{1}{2}\left(\frac{11}{4}\right)^2 + 6 = -\frac{121}{32} + \frac{192}{32} = \frac{71}{32}$

Example 1: Use a ~~Left~~ Trap Riemann Sum with **four** equal rectangles to *approximate* of the area of the region lying between the graph of $f(x) = \left(\frac{-1}{2}\right)x^2 + 6$ and the x-axis between $x = 1$ and $x = 3$.

1) Area ≈ __________ + __________ + __________ + __________

2) The base is _______. So the sub intervals are:
$\Delta x =$

[1,1.5], [1.5,2], [2,2.5], [2.5,3]



3) To find the first approximation use the ~~**left**~~ Trap **endpoints** as the heights of your rectangles.

$$\text{Trap} = \frac{1}{2}(b_1 + b_2)h$$

x	y
1	$\frac{11}{2}$
$\frac{3}{2}$	$\frac{39}{8}$
2	4
$2\frac{1}{2}$	$\frac{23}{8}$
3	$\frac{3}{2}$

$$\frac{1}{2}\left(\frac{11}{2}+\frac{39}{8}\right)\cdot\frac{1}{2} + \frac{1}{2}\left(\frac{39}{8}+4\right)\frac{1}{2} + \frac{1}{2}\left(4+\frac{23}{8}\right)\frac{1}{2} + \frac{1}{2}\left(\frac{23}{8}+\frac{3}{2}\right)\frac{1}{2}$$

$$\frac{1}{4}\left[\frac{11}{2}+\frac{39}{8}+\frac{39}{8}+4+4+\frac{23}{8}+\frac{23}{8}+\frac{3}{2}\right]$$

$$\frac{1}{4}\left[\frac{44}{8}+\frac{39}{8}+\frac{39}{8}+\frac{64}{8}+\frac{23}{8}+\frac{23}{8}+\frac{12}{8}\right] = \frac{244}{32}$$

$$= 7.32$$

$F(x) = -\frac{1}{2}x^2 + 6$

True area

$$\int_1^3 \left(-\frac{1}{2}x^2 + 6\right)dx = -\frac{1}{6}x^3 + 6x + C \Big[_1^3$$

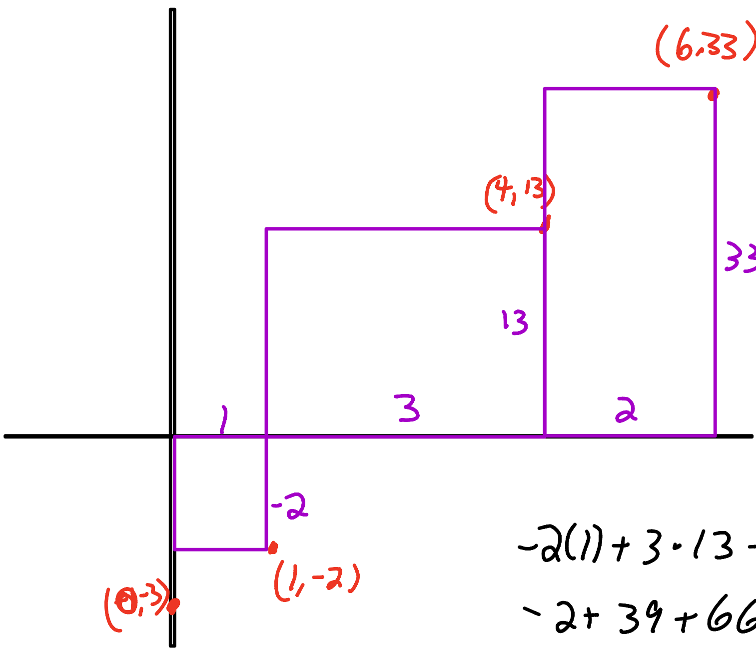
$$-\frac{1}{6}(3)^3 + 6(3) + C - \left[-\frac{1}{6}(1)^3 + 6(1) + C\right]$$

$$\frac{-27}{6} + 18 + \cancel{C} + \frac{1}{6} - 6 \cancel{- C} = \frac{-26}{6} + 12 = \frac{-13}{3} + 12$$

$$-4\frac{1}{3} + 12 = 7\frac{2}{3}$$

Example 3: For the function $f(x) = x^2 - 3$, $0 \le x \le 6$, partition the interval [0,6] into 3 subintervals [0,1],[1,4],[4,6] and form the Right Riemann sum.

$[x, y]$



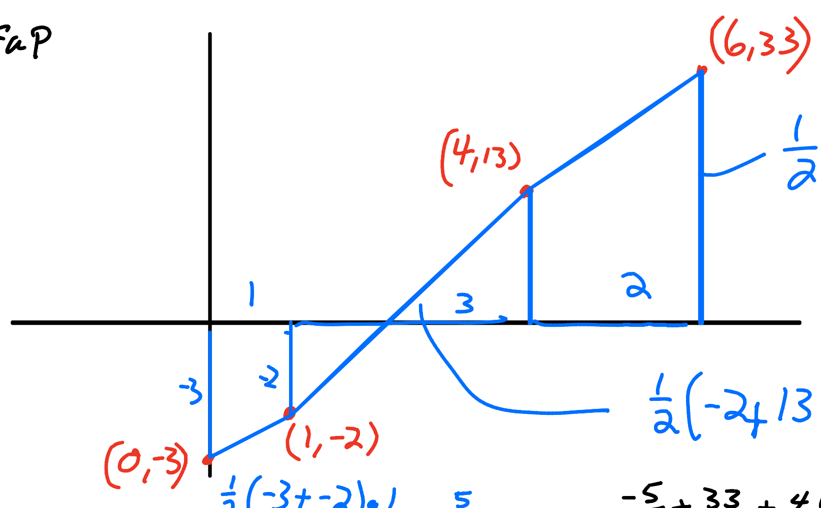
x	$y = x^2 - 3$
0	-3
1	-2
4	13
6	33

$-2(1) + 3 \cdot 13 + 2 \cdot 33$

$-2 + 39 + 66 = 103$

Example 3: For the function $f(x) = x^2 - 3$, $0 \le x \le 6$, partition the interval [0,6] into 3 subintervals [0,1],[1,4],[4,6] and form the ~~Right~~ Riemann sum.

Trap



$\frac{1}{2}(13+33) \cdot 2 = 46$

$\frac{1}{2}(-2+13) \cdot 3 = \frac{33}{2}$

$\frac{1}{2}(-3+-2) \cdot 1 = -\frac{5}{2}$

$-\frac{5}{2} + \frac{33}{2} + 46 = 14 + 46 = 60$

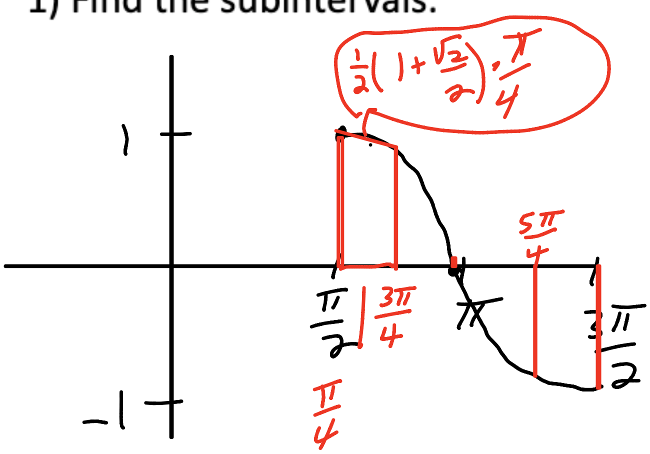
$$\int_0^6 (x^2-3)\,dx = \frac{1}{3}x^3 - 3x + c \Big]_0^6 = \frac{1}{3}(6)^3 - 3(6) + c - \left[\frac{1}{3}(0)^3 - 3(0) + c\right]$$

$$\frac{216}{3} - 18 \cancel{+c} - \cancel{0} + \cancel{0} \cancel{-c}$$

$$72 - 18 = 54$$

Example 4: Use the Trapezoidal Rule to approximate the area under $g(x) = \sin x$, between $[\underbrace{\frac{\pi}{2}, \frac{3\pi}{2}}_{\pi}]$, using $n = 4$. You may use a calculator.

1) Find the subintervals.



x	y
$\frac{\pi}{2}$	1
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$
π	0
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$\frac{3\pi}{2}$	-1

Example 5: *Approximate* $\int_1^7 (x-3)^2\,dx$ by partitioning into 3 sub intervals each of length 2 using Right Riemann sum.

Let $f(x) = (x-3)^2$

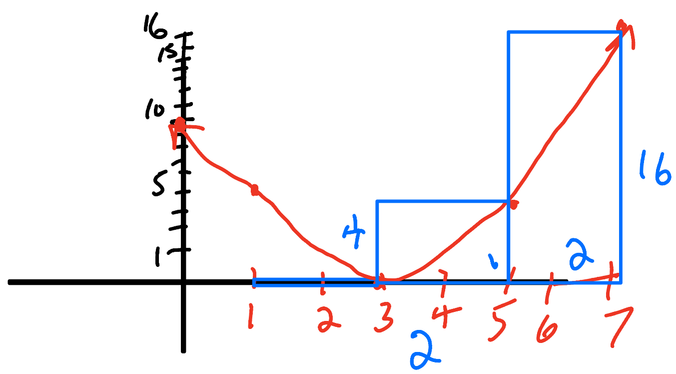
2) The base is _______. So the sub intervals are: $[1,3], [3,5], [5,7]$

3) To find the first approximation use the **right endpoints** as the heights of your rectangles.

$2(0) + 2(4) + 2(16)$

$8 + 32 = 40$

x	y
3	0
5	4
7	16



Riemann Sum with Tables

The table below displays the values of g for select times x

$\frac{6}{3}=2$

width=2

x	2	3	4	5	6	7	8	9	10	11
$g(x)$	20	17	16	14	10	6	5	3	2	1

(a) Estimate $\int_2^8 g(x)\,dx$ with 3 equal subintervals using Right Riemann Sum. Is your answer and over or underestimate. Justify.

x	y
4	16
6	10
8	5

$2(16)+2(10)+2(5)$

(b) Estimate $\int_3^9 g(x)\,dx$ with 2 equal sub intervals using Left Riemann Sum. Is your answer and over or underestimate. Justify.

$9-3=6$

x	y
3	17
6	10

$3\cdot 17+3\cdot 10$

$\frac{6}{2}=3$